New Cryptographic Scheme with Mellin Transformation

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Abstract

The aim of this research is to introduce/explore some new techniques for Encryption and Decryption of secret messages as privacy is the most important factor nowadays The process of encoding and decoding through Mellin transformation of any secret information, message and personal data is described by taking some appropriate functions. Mellin transformation is used for the phenomenon of improper integral. In this research special case of Taylor series, which is called Maclaurin series, is used for simple and exponential functions. Mellin transformation is applied after applying the special case of Taylor series on simple as well as exponential functions. Numeric values from 0 to 25 are assigned to alphabets from a to z and values 26 to 51 are allowed to A to Z capital alphabets. A new mathematical algorithm is used to Encrypt and Decrypt the confidential matters in which mod 63 is used for better calculations. Encryption and Decryption of personal data are done by using some tables. The use of thesetables is important for securing our system. The most important factor of this research is the generation of key, which is used in decoding process of specific information. This research is extension of previous research related to this topic by using mod 63 with Mellin transformation. This encryption will secure many aspects of our daily life such as ATM cards, computer passwords, and transmitting financial information, etc.

Keywords: Cryptography, Plaintext, Cipher text, Key, Encryption, Decryption.

Introduction

At present time, protection of privacy has become inevitable due to cyber crimes. Everyone wants to secure one's data, information, secret messages and personal networks. Mobile phones are the most commonly used device, and its users want to protect their phones with some password. * we find many websites which cannot be used due to the privacy factor.

(The above paragraph lacks cohesion and meaningful sense)

Cryptography is used for e-mail encryption. It is used to secure websites. During Second World War, it was used for breaking of code. An Encryption machine called German Enigma was used in Second World War. Cryptography is usually related to modern electronic communication. In ancient Greece, there was a device used for secret writing called Scytale of Sparta. Generally, Cryptography is known as the process of encoding and decoding of any secret information, but Cryptography can now be used for many other purposes. Cryptography is the combination of two Latin words 'Crypt' meaning 'secret' and 'Graphia' meaning 'writing'.

The old definition of Cryptography just focused on giving codes and was less beneficial. Nowadays, Cryptography encompasses much more. Now, Cryptography deals with algorithm for ensuring integrity and modern methods of exchanging hidden keys. Modern Cryptography gives more authentications to. In many books, foundation of Modern Cryptography is discussed in detail [10].

There is another interesting role of Cryptography, due to which it is called Modern Cryptography. (The role should be mentioned here) The other aspect of Modern Cryptography is GAMES. Cryptography is used in playing games, especially communication Games. Some communication games are being played for business, therefore, a fair playing is necessary which becomes possible due to Modern Cryptography. In these games, players playing from different countries need security. Some players use illegal ways to defeat other players, cleverly. The purpose of defeating some player is to harm someone and may be to achieve

some goal against the other player. Someplayers use tricks for their untitled advantages. Modern Cryptography allows a fair play without any fear [17].

Actually, Cryptography is the need to secure our communication and privacy of information which we send. People use many techniques for hacking personal data of someone, *Cryptographers have introduced many latest methods to secure information. This remarkable discipline is called modern Cryptography [3].

Inescapably, modern Cryptography is application of mathematical techniques used for securing digital personal information and our system against adversarial attacks. Modern Cryptography allows us to identify what types of threat are there and how can we save our messages. It tells us which Cryptographic scheme is beneficial [14].

Lakshmi explained Laplace Transformation [16] for Encryption [25] and Decryption of information by taking exponential function. Hiwarekar [13] elaborated/s Laplace Transformation for Cryptography with new method, in which he used Elementary function. There is a lot of work on Laplace Transformation presented by different Cryptographers with using many functions such as Exponential function, Trigonometric functions, Hyperbolic functions, gamma function, etc. In most of the papers, they changed function of different types and followed the same algorithm with minor changes.

Hiwarekar [12] explained Applications of Laplace Transformation in process of coding and decoding and for that purpose he used the Hyperbolic function. He used some standard results of Laplace Transformation. Someone introduced a new transformation [6] for Cryptography called Sumudu Transformation. Pandey and Som [18] told that Images can also be Encrypt using matrix transformation. To precede this process, novel Cryptosystem was used. Acharya [1] worked on polynomials like Chebyshev polynomials that how polynomials was used for coding and decoding. Interested readers may read [4, 5, 7, 9] and [22] for further interest.

Mellin transformation is an Integral transformation. In this paper, researcher followed a specific algorithm and some standard

tables for coding and decoding. Key, which was used, remained the same. We took Taylor series and then applied Sumudu Transformation for Encryption and Decryption of messages and also used results of Sumudu Transformation generated by applying this transformation on those specific functions. At the end the Cryptographer used only numerical values by neglecting variable to form code. Transformation was used to encrypt the specific messages and inverse of Sumudu Transformation was used for decryption.

Main Results

2.1) Mellin Transformation

The Mellin Transformation which I took is defined as [19]:

$$M[g(x);h] = \int_0^\infty g(x) (x)^{(h-1)} dx$$
 for $h > 0$, (1)

The inverse of Mellin Transformation can be defined as:

$$M^{-1}[g(x);h] = \int_{c+i\infty}^{c-i\infty} g(x) (x)^{(-h)} dx.$$
 (2)

2.2) Mellin Inversion Theorem

Mellin inversion theorem is actually based on a condition under which inverse of Mellin transformation [15] holds and get back to Mellin transformation under this condition which is a < Re(h) < b. This theorem can be stated as:

$$M^{-1}[g(x);h] = \int_{c+i\infty}^{c-i\infty} g(x) (x)^{(-h)} dx$$
 $a < Re(h) < b$. (3)

2.3) Taylor Series

The series expansion of any suitable function is known as Taylor series that can be defined as:

$$f(u) = f(u) + f^{1}(u)(x - u) + f^{2}(u)(x - u)^{2} + f^{3}(u)(x - u)^{3} + \dots + f^{n}(u)(x - u)^{n} + \dots$$
(4)

The derivative of chosen function is required. This series always takes about any arbitrary point. Maclaurin series is also Taylor series expansion taken about zero fixed point [11].

2.4) Some Theorems and Standard Results

There are some theorems used for Encryption and Decryption of personal messages. These theorems explain the methodology behind Encryption and Decryption of secret data.

2.5) Theorems for Encryption

Theorems for Encryption are used for converting plain text into cipher text.

2.5.1) Theorem No. 1

Given plaintext is in terms of $S_{a,0}$, where a = 1,2,3,4,..., under Mellin Transformation of any suitable function, then this function is multiplied by any coefficient for getting cipher text.

Proof

We prove, generally. by taking general values.

$$\begin{split} S_{a,1} &= 2^{2a+1} \, (2a+2)(2a+3) \, S_{a,0} \mod 63 \\ S_{a,1} &= Q_{a,1} - 63 K_{a,1} \quad \text{for} \qquad a = 0,1,2,3,4,... \\ &\qquad \qquad \text{where,} \\ Q_{a,1} &= 2^{2a+1} \, (2a+2)(2a+3) \, S_{a,0} \qquad \qquad a = 0,1,2,3,4,... \end{split}$$

and the key is defined as:

$$K_{a,1} = \frac{Q_{a,1} - S_{a,1}}{63}$$
 for $a = 0, 1, 2, 3, 4,...$

Here Q denotes quotient, S for denoting plaintext and K denotes key. We apply the same results to the output of this theorem in the next theorem and include two terms to get results however, Mellin transformation is different for different functions.

2.5.2) Theorem No. 2

Given plaintext is in terms of $S_{a,1}$, where a = 1,2,3,4,..., under Mellin transformation of any suitable function by $S_{a,0}$, then this function is multiplied by any coefficient for getting ciphertext.

Proof

On the same pattern this proof is done.

$$S_{a,2} = S_{a,1} \ 2^{2a+1} (2a+2)(2a+3) \quad \mod{63}$$

$$S_{a,2} = Q_{a,2} - 63K_{a,2} \quad \text{for} \quad a = 0,1,2,3,4,...$$
 where,
$$Q_{a,2} = S_{a,1} \ 2^{2a+1} (2a+2)(2a+3) \quad a = 0,1,2,3,4,...$$

and the key is defined as:

$$K_{a,2} = \frac{Q_{a,2} - S_{a,2}}{63}$$
 for $a = 0,1,2,3,4,...$

Here Q denotes quotient, S denotes plaintext and K denotes key. These formulas are used to find key and next terms. This theorem includes two terms.

In the next theorem we are applying operation on j time successively. We convert the plaintext into cipher text using the same key. This method is called iterative method and this process is called Encryption. (In the above paragraph, the researcher has mentioned next theorem on Encryption, but has not discussed.)

2.6) Theorems for Decryption

Any transformation which we use for Encryption and Decryption must have its inverse. I use (space bar issue, here) Mellin transformation which also has inverse [8], There is a theorem related to inverse of Mellin transformation called 'Mellin Inversion Theorem'. Mellin transformation is used for Encryption and its inverse is used for Decryption.

Some theorems related to Decryption of Mellin transformation are explained below and these are just statements which we use to solve our problem.

2.6.1) Theorem No. 3

With the help of given key $K_{a,0}$, where a=0,1,2,3,4,... given Ciphertext which is in the terms of $S_{a,1}$, a=0,1,2,3,4,... convert into the Plaintext $S_{a,0}$ by using inverse of Mellin transformation. Where,

$$2^{2a+1} (2a+2)(2a+3) S_{a,0} = 63K_{a,0} + S_{a,1}; \quad a = 0,1,2,3,4,...$$

$$S_{a,0} = \frac{63K_{a,0} + S_{a,1}}{2^{2a+1} (2a+2)(2a+3)}; \quad a = 0,1,2,3,4,...$$

and

$$Q_{a,0} = 63K_{a,0} + S_{a,1};$$
 $a = 0,1,2,3,4,...$

Here $S_{a,0}$ is a Plaintext and $S_{a,1}$ is the ciphertext. Decryption is also done by iterative method so next theorem is about successively j times iteration of the same procedure.

2.6.2) Theorem No. 4

With the help of given key $k_{a,j-1}$, where a, j=0,1,2,3,4,... given Ciphertext which is in the terms of $s_{a,j}$, a, j=0,1,2,3,4,... convert into the Plaintext $S_{a,j-1}$ by using inverse of

Mellin transformation.

Where.

$$\begin{split} 2^{2a+1} \left(2a+2 \right) & (2a+3) \, S_{a,j-1} = 63 K_{a,j-1} + \, S_{a,j} \; ; \quad a \; , \; j=0,1,2,3,4,\dots \\ S_{a,j-1} & = \quad \frac{63 K_{a,j-1} + \, S_{a,j}}{2^{2a+1} \left(2a+2 \right) \left(2a+3 \right)} \; \; ; \quad a \; , \; j=0,1,2,3,4,\dots \end{split}$$

And

$$Q_{a,j} = 63K_{a,j} + S_{a,j}$$
; $a, j = 0,1,2,3,4,...$

Here $S_{a,j-1}$ is a Plaintext and $S_{a,j}$ is the ciphertext.

2.7) Tables (used) for Encryption and Decryption

There are some standard tables used for Encryption and Decryption of personal data.

2.7.1) Tables for Encryption

The data which we want to Encrypt may include alphabets in small letters or in capital letters or may include digits. Conversion of plaintext into ciphertext is called Encryption. During Encryption of any personal messages some tables may be used according to the requirement of the message.

• The first table is of Alphabets in small letters.

A	В	С	D	Е	F	G	Н	i	J	K	L	M	n	О
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
P	Q	R	S	T	U	V	W	X	Y	Z				
15	16	17	18	19	20	21	22	23	24	25				

Table 1: Encryption Table

(Table#1 contains some CAPITAL letters, while researcher has mentioned only small letters for Table#1)

• The second table is about alphabets in capital letters.

A	В	C	D	Е	F	G	Н	I	J	K	L	M	N	О
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
P	Q	R	S	T	U	V	W	X	Y	Z				
41	42	43	44	45	46	47	48	49	50	51				

Table 2: Encryption Table 2

Digit 52 is used as space bar. In some secret messages there is space for example message as 'E prime', here after E there is space which is assigned 52 during encryption. In some messages, which we want to encode there are digits which also need some codes. For this purpose, there is another table given below:

0	1	2	3	4	5	6	7	8	9
53	54	55	56	57	58	59	60	61	62

Table 3: Encryption Table 3

Results and Discussions

3.1) Encryption Process

First of all, we have mentioned function and some results, then by using the results stated below we will

- The function which is used: $f(u) = \frac{1}{1-u}$.
- The series taken for this purpose is stated as

$$f(u) = f(u) + f^{(1)}(u)(x-u) + \frac{f^{(2)}(u)}{2!}(x-u)^2 + \frac{f^{(3)}(u)}{3!}(x-u)^3 + \dots + \frac{f^{(n)}(u)}{n!}(x-u)^n + \dots$$

• The Mellin Transformation used here is defined in Eq. (1). Now Encryption process will start under following steps

Step 1

The function is $f(u) = \frac{1}{1-u}$. We find the Taylor series which is

$$f(u) = f(u) + f^{(1)}(u)(x-u) + \frac{f^{(2)}(u)}{2!}(x-u)^2 + \frac{f^{(3)}(u)}{3!}(x-u)^3 + \dots (5)$$

All the required derivatives of the given function are performing below. Where f(u) denotes the selected function.

$$f^{(1)}(u) = (-1)(1-u)^{-2}(-1),$$

$$f^{(1)}(u) = \frac{1}{(1-u)^2},$$
(6)

where $f^{(1)}(u)$ denotes the first derivative.

$$f^{(2)}(u) = (-2)(1-u)^{-3}(-1),$$

$$f^{(2)}(u) = \frac{2}{(1-u)^3},$$
(7)

where $f^{(2)}(u)$ denotes the second derivative. Similarly,

$$f^{(3)}(u) = (-6)(1-u)^{-4}(-1),$$

$$f^{(3)}(u) = \frac{6}{(1-u)^4},$$
(8)

where $f^{(3)}(u)$ denotes the Third Derivative.

And so on, we can find derivative depends upon our calculations. Now put all these derivatives in Eq. (5) that will become

$$f(u) = \frac{1}{1-u} + \frac{1}{(1-u)^2} (x-u) + \frac{2}{2!(1-u)^3} (x-u)^2 + \dots$$
 (9)

After simplification, it becomes

$$f(u) = \frac{1}{1-u} + \frac{1}{(1-u)^2} (x-u) + \frac{1}{(1-u)^3} (x-u)^2 + \dots$$
 (10)

We discuss the special case of Taylor series by taking u = 0 in Eq. (10). Then, the Taylor series becomes:

$$f(u) = 1 + x + x^2 + x^3 + \dots$$
 (11)

The series form of Eq. (11) can be written as:

$$f(u) = \sum_{n=0}^{\infty} x^n \tag{12}$$

Now, we form a new function by multiplying the existing function with suitable coefficient.

As

$$\frac{1}{1-u} = f(u) = 1 + x + x^2 + x^3 + \dots$$

SO

$$\frac{1}{1-u}(x^2) = x^2 + x^3 + x^4 + x^5 + \dots$$
 (13)

The series form of Eq. (13) is written as:

$$\frac{1}{1-u}(x^2) = \sum_{n=0}^{\infty} x^{n+2} \tag{14}$$

Step 2

Now we start the process of Encryption. The message which is to be Encrypted is "We Love Math".

W	e	-	L	О	V	e	-	M	a	t	h	
---	---	---	---	---	---	---	---	---	---	---	---	--

Table 4: Message in Plaintext

We assign digits to these alphabets from tables 1, 2 and 3

W	Е	-	L	О	V	e	-	M	A	t	h
48	4	52	37	14	21	4	52	38	0	19	7

Table 5: Assigned Plaintext

	As Plaintext is in form of $S_{a,0}$ according to theorem, so each assigned digit of Plaintext write									
of Flamilext WII	ic .									
$S_{0,0} = 48$	$S_{1,0}=4$	$S_{2,0} = 52$	$S_{3,0} = 37$	$S_{4,0} = 14$						
$S_{5,0} = 21$	$S_{6,0} = 4$	$S_{7,0} = 52$	$S_{8,0} = 38$	$S_{9,0} = 0$						
$S_{10,0} = 19$	$S_{11,0} = 7$	$S_{n,0} = 0$	for $n \ge 12$.							

As new function becomes like $\frac{1}{1-u}(x^2)$,

where,

$$\frac{1}{1-u} = f(u) = 1 + x + x^2 + x^3 + \dots$$

Now multiply each Plaintext coefficient by each term of above equation respectively.

$S_{0,0}(1.x^0) = 48$	$S_{1,0}(x) = 4x S_2$	$2,0(x^2) = 52x^2$	$S_{3,0}(x^3) = 37x^3$
$S_{4,0}(x^4) = 14x^4$	$S_{5,0}(x^5) = 21x^5$	$S_{6,0}(x^6) = 4x^6$	$S_{7,0}(x^7) = 52x^7$
$S_{8,0}(x^8) = 38x^8$	$S_{9,0}(x^9) = 0$	$S_{10},_0(x^{10}) = 19x^{10}$	$S_{11,0}(x^{11}) = 7x^{11}$

Then multiply by x^2 ,

$48(x^2) = 48x^2$	$4x(x^2) = 4x^3$	$52x^2(x^2) = 52x^4$	$37x^3(x^2) = 37x^5$
$14x^4(x^2) = 14x^6$	$21x^5(x^2) = 21x^7$	$4x^6(x^2) = 4x^8$	$52x^7(x^2) = 52x^9$
$38x^8(x^2) = 38x^{10}$	$0(x^2)=0$	$19x^{10}(x^2) = 19x^{12}$	$7x^{11}(x^2) = 7x^{13}$

So we can write as

$$\frac{1}{1-u}(x^2) = S_{0,0}x^2 + S_{1,0}x^3 + S_{2,0}x^4 + S_{3,0}x^5 + S_{4,0}x^6 + S_{5,0}x^7 + S_{6,0}x^8 + S_{7,0}x^9 + S_{8,0}x^{10} + S_{9,0}x^{11} + S_{10,0}x^{12} + S_{11,0}x^{13} + S_{10,0}x^{11} + S_{10,0}x^{12} + S_{11,0}x^{13} + S_{10,0}x^{11} + S_{1$$

Now the new sum after adding coefficient x^2 is:

$$\frac{1}{1-u}(x^2) = \sum_{n=0}^{11} S_{n,0} x^{n+2}$$
 (15)

Putting values of Plaintext $S_{n,0}$ then it becomes

$$\frac{1}{1-u}(x^2) = 48x^2 + 4x^3 + 52x^4 + 37x^5 + 14x^6 + 21x^7 + 4x^8 + 52x^9 + 38x^{10} + 0 + 19x^{12} + 7x^{13}$$

Step 3

Now, we will apply Mellin transformation on both sides of the above equation.

$$M\left\{x^{n+2}\right\} = \frac{x^{n+h+2}}{n+h+2}, \qquad n = 0, 1, 2, ..., 11 \text{ and } h = 1$$

$$M\left\{\frac{1}{1-u}(x^2)\right\} = M\left\{48\left(\frac{x^5}{5}\right) + 4\left(\frac{x^6}{6}\right) + 52\left(\frac{x^7}{7}\right) + 37\left(\frac{x^8}{8}\right) + 14\left(\frac{x^9}{9}\right) + 21\left(\frac{x^{10}}{10}\right) + 4\left(\frac{x^{11}}{11}\right) + 52\left(\frac{x^{12}}{12}\right) + 38\left(\frac{x^{13}}{13}\right) + 0 + 19\left(\frac{x^{15}}{15}\right) + 7\left(\frac{x^{16}}{16}\right)\right\}$$

$$M\left\{\frac{1}{1-u}(x^2)\right\} = \{96e + (-1)\}x^5 + \{667e + (-3)\}x^6 + \{742e + (-2)\}x^7 + \{4625e + (-3)\}x^8 + \{155e + (-2)\}x^9 + \{21e + (-1)\}x^{10} + \{3636e + (-4)\}x^{11} + \{4727e + (-3)\}x^{12} + \{2923e + (-3)\}x^{13} + 01266e + (-3)\}x^{15} + \{4375e + (-4)\}x^{16}.$$

By taking value

$A_0 = 96$	$A_1 = 667$	$A_2 = 742$	$A_3 = 4625$
$A_4 = 155$	$A_5 = 21$	$A_6 = 3636$	$A_7 = 4727$
$A_8 = 2923$	$A_9 = 0$	$A_{10} = 1266$	$A_{11} = 4375$

According to the tables 1, 2 and 3 we take mod 63 all of these values. In general we can write as:

$$A_n = 63(Q) + R$$
 where $n = 0.1, 2, 3, \dots, 11$.

Here Q denotes the Quotient which forms Key and R is Remainder, forms the Ciphertext $S_{0,n}$ and $A_n = 63(Q) + R$ where n = 0, 1, 2, 3, ..., 11.

96 = 63(1)+33	mod 63	Where $A_0 = 96$ $K =$	1 $S_{0,0} = 33$
667 = 63(10)+37	mod 63	Where $A_1 = 667$ $K = 667$	$= 10 S_{0,1} = 37$
742 = 63(11)+49	mod 63	Where $A_2 = 742$ $K =$	11 $S_{0,2} = 49$
4625 = 63(73)+26	mod 63	Where $A_3 = 4625$ $K =$	73 $S_{0,3} = 26$
155 = 63(2)+29	mod 63	Where $A_4 = 155$ $K = 2$	$S_{0,4} = 29$
21 = 63(0)+21	mod 63	Where $A_5 = 21$ $K = 0$	$S_{0,5} = 21$
3636 = 63(57)+45	mod 63	Where $A_6 = 3636$ $K =$	$S_{0,6} = 45$
4727 = 63(75)+2	mod 63	Where $A_7 = 4727$ $K =$	75 $S_{0,7} = 2$
2923 = 63(46)+25	mod 63	Where $A_8 = 2923$ $K = 4$	$S_{0,8} = 25$
0 = 63(0) + 0	mod 63	Where $A_9 = 0$ $K =$	$0 S_{0,9} = 0$
1266 = 63(20)+6	mod 63	Where $A_{10} = 1266 \ K = 2$	$20 S_{0,10} = 6$
4375 = 63(69)+28	mod 63	Where $A_{11} = 4375$ $K =$	$= 69 S_{0,11} = 28$

The Key formed as:

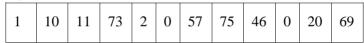


Table 6: Key of Coded Message

Remainder values assigned to $S_{0,n}$ that forms the Ciphertext.

$S_{0,0} = 33$	$S_{0,1} = 37$	$S_{0,2} = 49$	$S_{0,3}=26$	$S_{0,4} = 29$
$S_{0,5} = 21$	$S_{0,6} = 45$	$S_{0,7} = 2$	$S_{0,8} = 25$	$S_{0,9} = 0$
$S_{0,10} = 6$	$S_{0,11} = 28$	$S_{0,n}=0$		for $n \ge 12$.

Hence CODED MESSAGE is:



Table 7: Cipher text

So the Plaintext "We Love Math" is now converted into Cipher text "HLXADvTczagC".

3.2) Decryption Process

We can decrypt this message as well by putting back steps using inverse of Mellin transformation, which is also defined in detail by theorem called Mellin Inversion theorem.

Conclusion

The main purpose of Cryptography is to secure the system as protection(of privacy) is the most important factor nowadays. Mathematicians gave some beneficial algorithms by using some techniques for the security of personal data. We used Mellin transformation with special case of Taylor series for this purpose. The main results of this research are:

- This process covered space bar also for Encryption.
- Use of Mellin Transformation made calculations easy and simplest.
- Special case of Taylor series made simplest form of function.

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